TeX's DVI file preamble: deriving the *num* and *den* values

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In the documentation to the DVITYPE program it notes that the preamble of a DVI file contains basic information about the file as a whole. There are six parameters:

i[1] *num* [4] *den* [4] *mag* [4] *k*[1] *x*[*k*].

The *i* byte identifies the DVI format being used:

- $i = 2$ for standard DVI files

Notes:

- $i = 5$ for extended DVI files (XDV) produced by XeTeX—which is *not*, currently supported by the DVITYPE program.
- Here we are not discussing *mag*, *k* and *x.*

From the DVITYPE program documentation:

> The two parameters, *num* and *den*, are positive integers that define the units of measurement; they are the numerator and denominator of a fraction by which all dimensions in the DVI file could be multiplied in order to get lengths in units of 10^{-7} meters. For example, there are exactly 7227 TEX points in 254 centimeters, and TEX82 works with scaled points where there are 2^{16} sp in a point, so TEX82 sets:
>
> $num = 25400000$
> $den = 7227 \times 2^{16} = 473628672$

Deriving num and den

This short example shows how the *num* and *den* values are calculated. Let us firstly define the following abbreviations for various units:

- [tp] = TeX Point
- [sp] = TeX Special Point
- [mm] = millimetres
- [m] = metres

We know that the following equalities are true:

- $72.27[\text{tp}] = 1 \text{ inch} = 25.4[\text{mm}]$
- $65536[\text{sp}] = 1[\text{tp}]$ or, using $65536 = 2^{16}$, we can say $2^{16}[\text{sp}] = 1[\text{tp}]$
- $1[\text{mm}] = 1 \times 10^{-3}[\text{m}]$

According to the requirements noted by Knuth, we want to calculate some value, *f*, such that we can convert units of [sp] to some multiple of 10^{-7}[m]. In essence:

$1[\text{sp}] = f \times 10^{-7}[\text{m}]$

where *f* can be represented by a ratio of integers: a fraction $f = \frac{num}{den}$

We want to find the integers *num* and *den* and can proceed by using the above equalities and definitions of units:

Starting with $2^{16}[\mathrm{sp}] \;=\; 1[\mathrm{tp}]$ and $72.27[\mathrm{tp}] = 25.4[\mathrm{mm}]$

Firstly, using $72.27[\mathrm{tp}] = 25.4[\mathrm{mm}]$ and multiplying by 100 to convert to *integers*:

$7227[\mathrm{tp}] = 2540[\mathrm{mm}]$

Substituting $1[\mathrm{mm}] = 1 \times 10^{-3}[\mathrm{m}]$ we get

$7227[\mathrm{tp}] = 2540 \times 10^{-3}[\mathrm{m}]$

Using $2^{16}[\mathrm{sp}] \;=\; 1[\mathrm{tp}]$ we arrive at

$7227 \times 2^{16}[\mathrm{sp}] = 2540 \times 10^{-3}[\mathrm{m}]$

Knuth noted the desire to work in units of $10^{-7}[\mathrm{m}]$ so we simply divide both sides by 10^4:

$$\frac{7227 \times 2^{16}}{10^4}[\mathrm{sp}] = \frac{2540 \times 10^{-3}}{10^4}[\mathrm{m}]$$

Thus, using $\frac{10^{-3}}{10^4} = 10^{-7}$ we get

$$\frac{7227 \times 2^{16}}{10^4}[\mathrm{sp}] = 2540 \times 10^{-7}[\mathrm{m}]$$

Yielding:

$$1[\mathrm{sp}] = \frac{2540 \times 10^4}{7227 \times 2^{16}} \times 10^{-7}[\mathrm{m}]$$

or, expressed in integers,

$$1[\mathrm{sp}] = \frac{25400000}{473628672} \times 10^{-7}[\mathrm{m}]$$

Going back to our original question to find *f* such that

$1[\mathrm{sp}] = f \times 10^{-7}[\mathrm{m}]$

where *f* can be represented by a fraction: $f = \frac{num}{den}$, we can see that $num = 25400000$ and $den = 473628672$